Dedekind Domains

Introduction to Algebraic-Geometric Codes. Fall 2019

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Definition

A domain A is called a Dedekind domain if

1. A is noetherian;
2. $\dim(A) = 1$; and
3. A is integrally closed.

Example

Every PID is a Dedekind domain.

Theorem

Let A be a Dedekind domain with field of fractions K. Let L be a finite separable extension of K. Then B, the integral closure of A in L is a Dedekind domain.

Proof.

We proved that:

- The integral closure is integrally closed.
- A noetherian $\implies$ B noetherian.
- $\dim(A) = 1 \implies \dim(B) = 1$.

□

Recall a result we stated without a proof:

Proposition

Noetherian + Dimension 1 + UFD = PID

B may not be a UFD even if A is. Thus, unique factorization is not inherent in our extensions. However, we will later prove that unique factorization of ideals is! We'll show that

Theorem

Noetherian + Dimension 1 + UFI = Dedekind domain