Introduction to Algebraic-Geometric Codes **Spring 2019**

Exercise 5

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Exercise 5.1. *Let A be a domain and $\langle 0 \rangle \neq I \in Spec(A)$.*

(a) Assume A is noetherian. Prove or disprove:

(i) A/I is noetharian.

(ii) A_I is noetharian.

(b) Assume A is integrally closed. Prove or disprove:

(i) A/I is integrally closed.

(ii) A_I is integrally closed.

(c) Assume $\dim(A) = k$.

(i) What can you say about $\dim(A/I)$? Prove your statement.

(ii) What can you say about $\dim(A_I)$? Prove your statement.

(iii) What can you say about $\dim(A/I) + \dim(A_I)$? Prove your statement.

(d) According to all of the above, assume A is a Dedekind domain. Prove or disprove:

(i) A/I is a Dedekind domain.

(ii) A_I is a Dedekind domain.

Exercise 5.2. *Let A be a PID. Prove that $\dim(A[x]) = 2$.*

Exercise 5.3. *Let K be a field and let $f \in \mathsf{K}[x, y]$ be an irreducible polynomial.*

(a) Assume that K is algebraically closed, what is the dimension of $\mathsf{K}[x, y]/\langle f \rangle$?

(b) Let $P \in Max(\overline{\mathsf{K}}[x, y])$. Show that $P \cap \mathsf{K}[x, y]$ is maximal.

(c) What is the dimension of $\mathsf{K}[x, y]/\langle f \rangle$?