Course Summary

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Overview

Material we covered

The basic linear-algebraic theorems

1. The spectral theorem
2. Courant-Fischer
3. Perron-Frobenius
4. Cauchy's interlacing theorem

Material we covered

Basic SGT results

1. Graph drawing
2. Hoffman's bound on the chromatic number
3. Spectrum of random graphs
4. Cayley graphs and Paley graphs
5. Cheeger's inequality
6. Random walks on graphs

Material we covered

More advanced SGT results

1. Graphs as resister networks
2. The matrix-tree theorem
3. Spectral sparsification

Material we covered

Expander graphs

1. Basic properties: the expander mixing lemma, hitting property
2. Algebraic construction - the Gaber-Galil expander
3. Combinatorial constructions via the Zig-Zag product
4. Explicit near-Ramanujan expanders via the wide replacement product

Material we covered

Applications

1. Randomness extractors via expander random walks
2. Error reduction via expander random walks
3. Reingold's **SL** = **L**
4. Small-bias sets constructions

Future exploration

1. Directed graphs
2. Algorithmic aspects of SGT. In particular, spectral sparsification in nearly-linear time.
3. Many applications, e.g., for the max flow problem.
4. Recent applications to space-bounded derandomization
5. Special families of graphs (e.g., planar graphs)
6. High order Cheeger
7. Explicit bipartite Ramanujan graphs
8. Much more to say about expanders
9. High dimensional expanders

Thank you!

Thank you for tuning in, asking questions, generating interesting discussions, and raising great comments and suggestions!

Shir and Gil