Topics in Coding Theory: Locallity and Interaction **Winter 2020**

Exercise 3

Publish Date: 16 December 19 *Due Date: 30 December 19*

Exercise 3.1. *Let $\mathcal{C} : \mathbb{F}_q^k \to \mathbb{F}_q^n$ be a linear $(r, \delta, \sigma)-$locally correctable code. Show that there is $\mathcal{C}' : \mathbb{F}_q^k \to \mathbb{F}_q^n$, that is a linear $(r, \delta, \sigma)-$locally decodable code such that $Im(\mathcal{C}) = Im(\mathcal{C}')$.*

Exercise 3.2. *Let $RM(q, d, n)$ be the Reed-Muller code, as studied in class. In this question we want you to analyze the local correctabilty parameters (r, δ, σ), with respect to the following correcting algorithm.*

Let $n \geq 3$. Given an evaluation of a polynomial F corrupted in up to δ fraction of coordinates and a point $w \in \mathbb{F}_q^n$ the local corrector picks vectors $v_1, v_2 \in \mathbb{F}_q^n$ uniformly at random and considers a plane $M = \{w + \lambda v_1 + \mu v_2 \mid \lambda, \mu \in \mathbb{F}_q\}$ through w. The corrector queries coordinates of the evaluation vector corresponding to points in $M \setminus \{w\}$. Next the corrector computes a two- variable polynomial h that agrees with F on the biggest fraction of values, and outputs $h(0, 0)$.

Exercise 3.3. *Let $q = 2^l$ and $\Lambda \subset \mathbb{F}_q[x, y, z]$ be the set of all polynomials that when projected on a line, are of degree less then $q - 1$. Namely:*

$$\Lambda = \{f \in \mathbb{F}_q[x, y, z] \mid \forall A, B, C, a, b, c \in \mathbb{F}_q. \deg(f(At + a, Bt + b, Ct + c)) \leq q - 2\}$$

Note that Λ is a vector space over $\mathbb{F}$. Give a lower bound on the dimension of Λ.

Exercise 3.4. *Let A be a ring.*

(a) *Assume A is a finite domain (i.e there are no zero dividers in A). Show that A is a field.*

(b) *Prove or Disprove: Every two non parallel lines in A have a single intersection (a line in A is a function of the form $l(t) = at + b : A \to A$ where $a, b \in A$).*

(c) *Let I be the set of all zero dividers in A. Prove or Disprove: I is an ideal.*

Exercise 3.5. *Let $P(x_1, \ldots, x_n) \in \mathbb{F}[x_1, \ldots, x_n]$, $a, b \in \mathbb{F}^n$ and $i \in \mathbb{N}^n$.*

(a) *Prove that $mult(P, a) \leq wt(i) + mult(P^{(i)}, a)$.*

(b) *Let $P_{a,b}(t)$ be the polynomial $P(a+t{\cdot}b) \in \mathbb{F}[t]$. Show that for any $\lambda \in \mathbb{F}$, $mult(P_{a,b}, \lambda) \geq mult(P, a+\lambda{\cdot}b)$.*