Algebraic Geometry for Theoretical Computer Science

Assignment 7

Lecturer: Gil Cohen *Hand in date: December 18, 2014*

Instructions: Please write your solutions in LaTeX / Word or exquisite handwriting. Submission can be done individually or in pairs.

1. Let $\mathcal{O}$ be a valuating ring of the rational function field $K(X)/K$. Assume that $x \notin \mathcal{O}$. Show that in this case, $\mathcal{O} = \mathcal{O}_\infty$. This completes the proof done in class, and shows that we have accounted for all places of the rational function field.
2. Let A, A' be two divisors of F/K such that $A \sim A'$. Show that $\mathcal{L}(A)$ and $\mathcal{L}(A')$ are isomorphic as vector spaces over K.
3. Let F/K be a function field. Show that $\mathcal{L}(0) = K$.
4. Let A be a divisor of a function field. Show that if $A < 0$ then $\mathcal{L}(A) = \{0\}$.
5. Let A be a degree zero divisor of a function field. Prove that the following assertions are equivalent.
   (a) A is principle.
   (b) $\ell(A) \geq 1$.
   (c) $\ell(A) = 1$.