**Topics in Coding Theory: Locality and Interaction** **Winter 2020**

# Exercise 2

*Publish Date: November 26, 2019* *Due Date: December 16, 2019*

**Exercise 2.1** *Let $p$ be a prime number and $k \geq 1$. How many pairs $x, y \in \mathbb{F}_{p^k}$ satisfy $Tr(y) = N(x)$?*

**Exercise 2.2** *Let $\pi$ be an interactive coding scheme with distance $\delta$ and rate $R$, that uses alphabet of size $c \in \mathbb{N}$. Describe a procedure that converts $\pi$ into a new scheme $\pi'$ that uses the alphabet $\{0, 1\}$. What is the best distance and rate of $\pi'$ that you can achieve, as a function of $\delta, R, c$?*

**Exercise 2.3** *Describe an interactive coding scheme $IC(\epsilon, \pi)$ such that for every interactive protocol $\pi$ of depth $n$ and a small enough constant $\epsilon$ it holds that:*

$$\frac{n}{depth(IC(\epsilon, \pi))} = n^{-O_\epsilon(1)}.$$

**Exercise 2.4** *Describe an explicit tree code construction satisfying that there exists a constant $c \in \mathbb{N}$ such that at depth $n$ one is allowed to use $n^c$ colors.*

**Exercise 2.5** *Show that there is a function $f(x, y)$ such that for every interactive protocol $\pi$ that computes $f$ there exists an adversary $A$ that can corrupt $\frac{1}{4}$ of the communication, and inputs $x_0, y_0$ such that $A$ can force $\pi(x_0, y_0) \neq f(x_0, y_0)$. You can assume that $\pi$ is alternating, and in each turn one bit is being sent.*

**Exercise 2.6** *In this question we describe the model of channels with feedback. Here we assume that in addition to the noisy (main) channel, Alice and Bob share another noiseless (feedback) channel. However, the additional channel can be used only in a very restricted manner: when a symbol $\sigma$ is communicated through the noisy channel and a symbol $\sigma'$ (possibly corrupted) is received, the symbol $\sigma'$ is then automatically sent over the feedback channel back to the sender. This way, the sender can learn whether his symbol was correctly received at the other side and, if not, what symbol was received. As before, the main channel can be subjected to adversarial noise.*

1. *Describe an efficient interactive coding scheme for this model, that is resilient to an error rate of $\frac{1}{4}$. You may use any constant size alphabet.*
2. *Show how to reduce the alphabet to $\{0, 1\}$. What is the maximal error rate your scheme can tolerate?*