Introduction to Algebraic-Geometric Codes **Spring 2019**

Exercise 4

Publish Date: 11 April 19 *Due Date: 02 May 19*

Exercise 4.1. (a) *Find a primitive element (a generator) in the separable extension* $L = \mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$ *over* $\mathbb{Q}$.

(b) *Describe all the embeddings of* L *in* $\overline{\mathbb{Q}}$ *over* $\mathbb{Q}$.

(c) *Let* $\mathbb{F}$ *be a field and* $\alpha \in \overline{\mathbb{F}}$ *be a separable element. Show that the embeddings of* $\mathbb{F}(\alpha)$ *in* $\overline{\mathbb{F}}$, $\Gamma_{\mathbb{F}(\alpha)/\mathbb{F}} = \{\sigma_1, \ldots, \sigma_n\}$, *are linearly independent.*

Exercise 4.2. *Let* $\mathbb{F}$ *be a field, show that the extension* $\overline{\mathbb{F}}[x_1, \ldots, x_n]$ *over* $\mathbb{F}[x_1, \ldots, x_n]$ *is integral.*

Exercise 4.3. *Let* A *be a domain, and let* B *be an integral extension of* A. *Prove that* B *is a field if and only if* A *is a field.*

Exercise 4.4. *Let* K *be a field and let* $f(x,y) = y^2 - x^3(1-x), g(x,y) = x+y+1, h(x,y) = y^3 - x \in \overline{K}[x,y]$.

Prove or disprove:

1. C_g *is integrally closed in* $\overline{K}(Z_g)$.
2. C_f *is integrally closed in* $\overline{K}(Z_f)$.
3. C_h *is integrally closed in* $\overline{K}(Z_h)$.