

Exercise 3: Circuits and Entanglement

As promised, this exercise comes marked with the questions that should be dealt with before this week's recitation. These are marked by a box around the labels (there are 3 of those in this exercise: Circuits-1,2, Entanglement-1). The unboxed questions should still be dealt with, but we may only see them at the NEXT recitation (in a week and a half).

3.1 Circuits

1. Calculate and describe what the circuit in Figure 3.1 does.

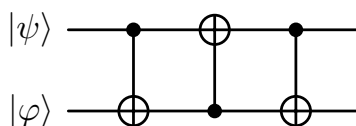


Figure 3.1: Circuit on 2 qubits. the gates are all CNOT, where the $\bullet$ is the control qubit and the $\oplus$ is the target qubit.

2. Implement a measurement in some orthonormal basis $\{|v_i\rangle\}_{i=0}^{d-1}$ using unitary transformations and computational-basis measurements.
3. Revisiting the “Troubling Experiments” from unit 2 by circuits:
Recall Experiments 2,3,4, where we had a ”hardness” splitting device, and we merged the two paths into a ”colour” or ”hardness” splitting devices which we used to measure the final colour/hardness of the photon. Recall that hardness is actually talking about the Hadamard (or the X) basis $\{|+\rangle, |-\rangle\}$ for the qubit, while colour is about the computational (or the Z) basis $\{|0\rangle, |1\rangle\}$. Recall also the two possible experiments from Experiment 4: either both paths are open, or we block one of them.

- (a) Model the "which path" information as another qubit and draw the circuits corresponding to experiments 2,3,4. For experiment 4, we would like to model both the open-paths version and the one-blocked-path version (by two separate circuits), but modelling a blocked path will prove difficult (why?). Instead, model a version where we measure which path the photon takes instead of blocking one of them (before merging the paths).
- (b) Calculate the overall state after each gate in the circuits.
- (c) Calculate the probabilities of the final measurement's outcomes for each circuit.

3.2 Entanglement and Perfect Correlations

1. Suppose we measure the first qubit of an EPR pair $|EPR\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ in a basis $\{|u\rangle, |u^\perp\rangle\}$.
 - (a) What is the probability of each outcome?
 - (b) Assume the measurement result was $|u\rangle$. What is the state of the second qubit after the measurement?
 - (c) What changes if we assume $|u\rangle$ is a real vector (in the computational basis), i.e. $|u\rangle = a|0\rangle + b|1\rangle$ for real $a, b \in \mathbb{R}$?
2. (a) Let U be a 2×2 unitary and let $|\Psi^-\rangle$ be the singlet state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Compute $(U \otimes U)|\Psi^-\rangle$. (Namely, apply U to each qubit of $|\Psi^-\rangle$.)
 - (b) Use the previous subquestion to show that if Alice and Bob share a singlet state, Alice can perform an operation on her qubit that is equivalent to Bob performing a unitary U of her choice on his qubit (without communicating with Bob or acting on his qubit).