Introduction

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Overview

What is this seminar about?

- Spectral graph theory is the study of graphs by their eigenvalues, namely, their "spectrum", and less typically also their eigenvectors.
- Ramanujan graphs have the "best" spectrum.
- They have little to do with Ramanujan the mathematician.

This is a mathematical seminar in nature that combines:

- graph theory and combinatorics,
- linear algebra,
- probability,
- real polynomials,
- and a bit of other stuff, e.g., groups.

It is insanely beautiful yet very challenging.

About us

I am a theoretical computer scientist

Technion → Weizmann
→ Caltech
→ Princeton
→ Tel Aviv University

My interests are

1. Complexity theory, especially randomness related questions
2. Explicit constructions
3. Coding theory
4. Spectral graph theory (SGT)

Gal (galmaor@mail.tau.ac.il) is my PhD student, working mostly within SGT. You can reach me at coheng@gmail.com.

Course mechanics

1. We meet physically on Mondays 9-11 at Orenstein Building, room 110.
2. `www.gilcohen.org/2020-ramanujan`.
3. Tentatively, presentations are in pairs using slides (not board).
4. Each student should give one of the two hours.
5. I will ask questions both to measure understanding and for clarification.
6. I will give (at least) three lectures.
7. You are highly encouraged to meet with me and Gal the week prior to your talk to go over it and get some feedback (which will not affect your grade).
8. You are welcome to ask questions throughout the semester (email me for setting a short appointment).

Grade and participation

The grade will be determined by

1. The quality of the lecture and level of understanding (students in a pair may get different grades).
2. Participation during the talks.

Grade and participation

All students are required to physically attend the seminar. However,

1. A student may choose not to attend one lecture (though not a lecture given by me). Missing two or more lectures will affect the grade.
2. In special circumstances (e.g., bidud), and only given my approval in advance, a student may attend via Zoom with an open camera.

Overview

Tips

General tips:

1. Understand everything you are asked to read to the deepest level you possibly can.
2. Read a bit more to get the context.
3. Digest with your partner.
4. Practice your talk prior to the meeting at least twice.
5. Make use of our pre-talk feedback.
6. Welcome questions during your talk.

Learning outcomes

1. Get a taste of SGT.
2. Learn an extremely important and deep result that combines much of what you have learned.
3. Practice presenting challenging material.

Overview

The seminar's structure

The general (somewhat tentative) structure of the seminar is

1. Introduction and motivation
2. Expanders
3. Ramanujan graphs and their relaxations

We mostly follow part of the (online) book by Spielman (`http://cs-www.cs.yale.edu/homes/spielman/sagt/sagt.pdf`).

The seminar's structure

Part 1 - Introduction and motivation.

1. Introduction to SGT, expanders and Ramanujan graphs (given by me).
2. Introductory continued (given by me).
3. Expander codes (part of chapter 28; chapter 29).
4. The Alon-Boppana bound (short paper), eigenvalue interlacing, and a bound on the chromatic number (part of chapter 4).

If you want to get a more solid introduction to SGT, I will be happy to share the video lectures of my course from last year (email me and ask). The course homepage is `https://www.gilcohen.org/2020-spectral-graph-theory`.

The seminar's structure

Part 2 - Expanders and spectral sparsification.

5. Explicit construction of expanders via the line graph (chapter 30).
6. Spectral sparsifiers (chapter 33).
7. Bilu-Linial expanders (part of the original paper).

The seminar's structure

Part 3 - Ramanujan graphs and their relaxations

8. Preparation for this part (by me).
9. The expected characteristic polynomial (chapter 42).
10. Quadrature for the finite free convolution (chapter 43).
11. Half Ramanujan graphs of every size and every degree (chapter 44).
12. Matching polynomials of graphs (chapter 45).
13. Bipartite Ramanujan graphs of every degree (chapter 46).

Overview

What is next?

1. In the first two lectures I will cover the basics of SGT. In particular, we will motivate and define Ramanujan graphs.
2. During this time, by the end of next Thursday (October 21) you are asked to pair and choose a lecture from the homepage.
3. Each pair will email me (`coheng@gmail.com`) its first, second and third choice by then, and we will try to accommodate your requests. There are no guarantees.
4. I will need "volunteers" for the third lecture (on expander codes). First come first served.